$$u''(s) + K(s)\cdot u(s) = 0 \quad *$$

linear!

Stepwise solution, constant K

↳ steps $1 \dots n \dots$

$$\begin{pmatrix} u \\ u' \end{pmatrix}_{n+1} = M_n \begin{pmatrix} u \\ u' \end{pmatrix}_n$$

„Cascade"

$$\begin{pmatrix} u \\ u' \end{pmatrix}_{n+1} = M_n \cdot M_{n-1} \cdot M_{n-2} \cdots M_0 \begin{pmatrix} u \\ u' \end{pmatrix}_0$$

general form

$$M_n = \begin{pmatrix} \cos(\sqrt{K_n}\cdot L_n) & \frac{1}{\sqrt{K_n}}\sin(\;) \\ -\sqrt{K_n}\sin(\;) & \cos(\;) \end{pmatrix}$$

L_n = length of segm. „n"

K_n : { drift, dipole, ⋮

↳ used in tracking codes

go to „infinitely small steps"!

$$M_{0\to s} = \begin{pmatrix} f_1(s) & f_2(s) \\ f_3(s) & f_4(s) \end{pmatrix}$$

coefficients as functions of s!

another view! * Diff. eq. second order

↳ two fundamental solutions.

(remember K = const. $u = a \cdot \cos() + b \cdot \sin()$

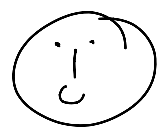
$K = K(s)$

$$u(s) = a \cdot C(s) + b \cdot S(s)$$

$$\rightarrow \quad u'(s) = a \cdot C'(s) + b \cdot S'(s)$$

!

$$M_{0\rightarrow s} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}$$



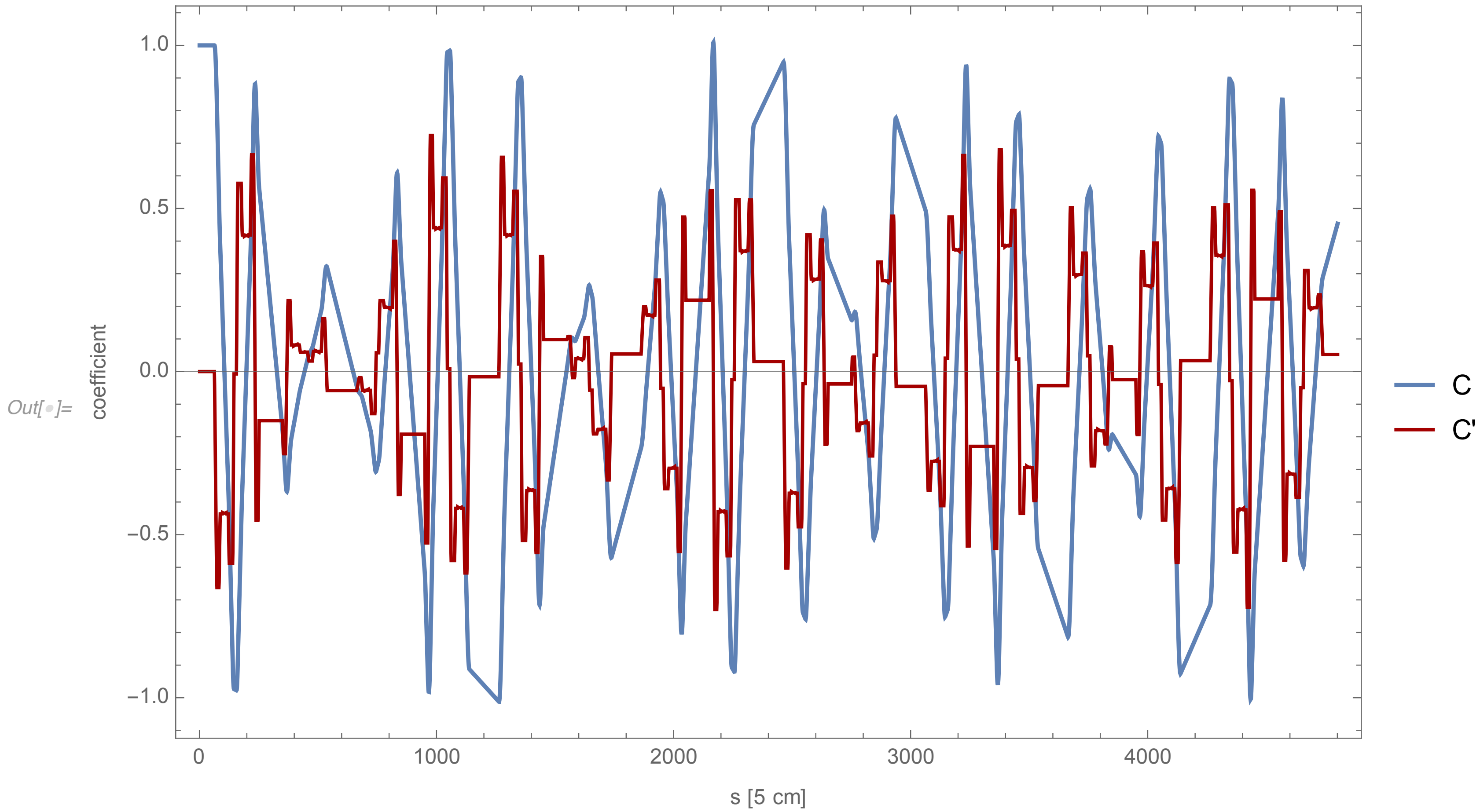
"CS-matrix"

derivatives! two functions only.

notice: | C, S are not cos or sin functions!
| C, S depend on where $s = 0$! $(M_{0\rightarrow s})$

$C, S \;\hat{=}\;$ particle trajectories $\begin{cases} \text{config. space} & (x,s), (y,s) \\ \text{phase space} & (x,x'), (y,y') \end{cases}$

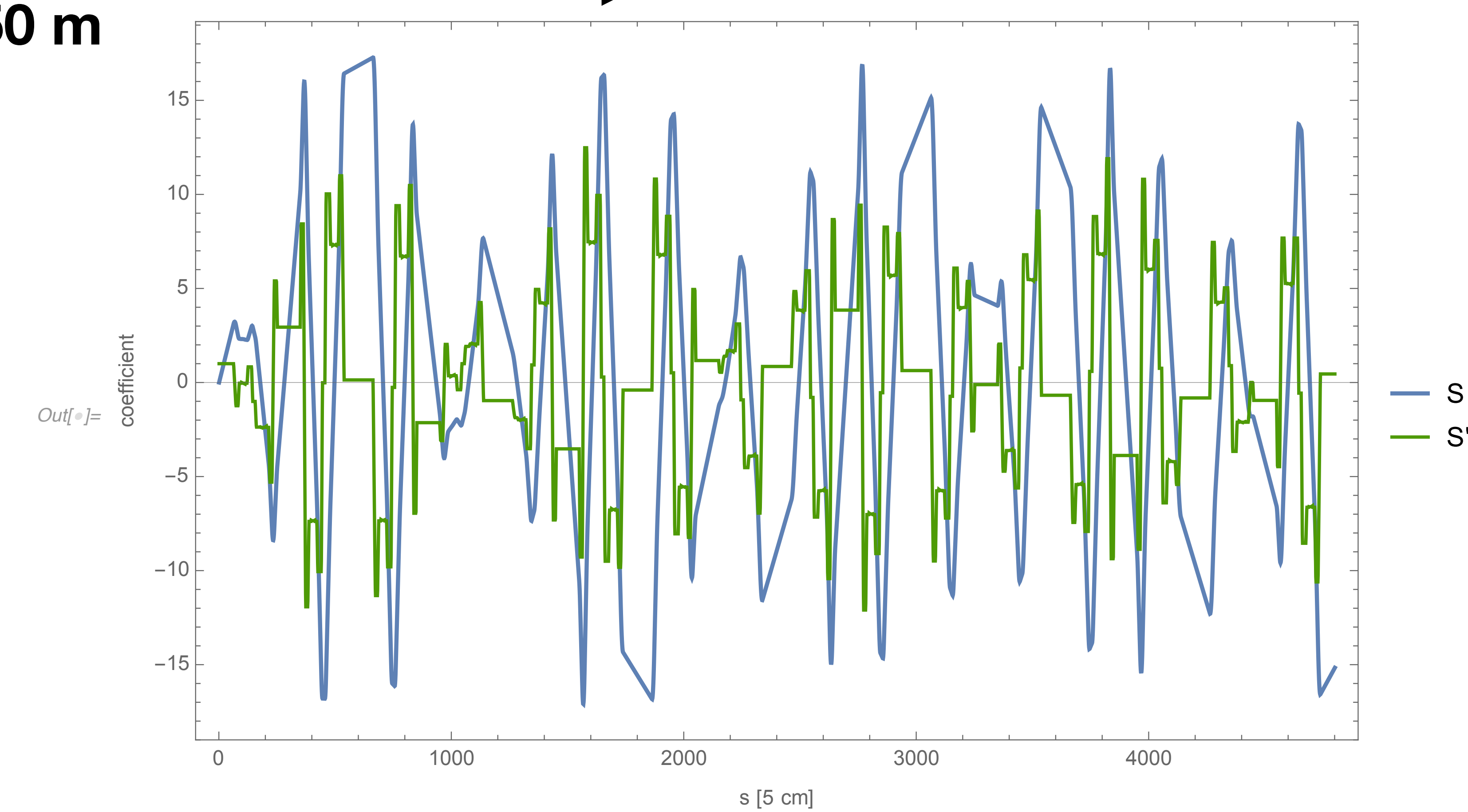
key note

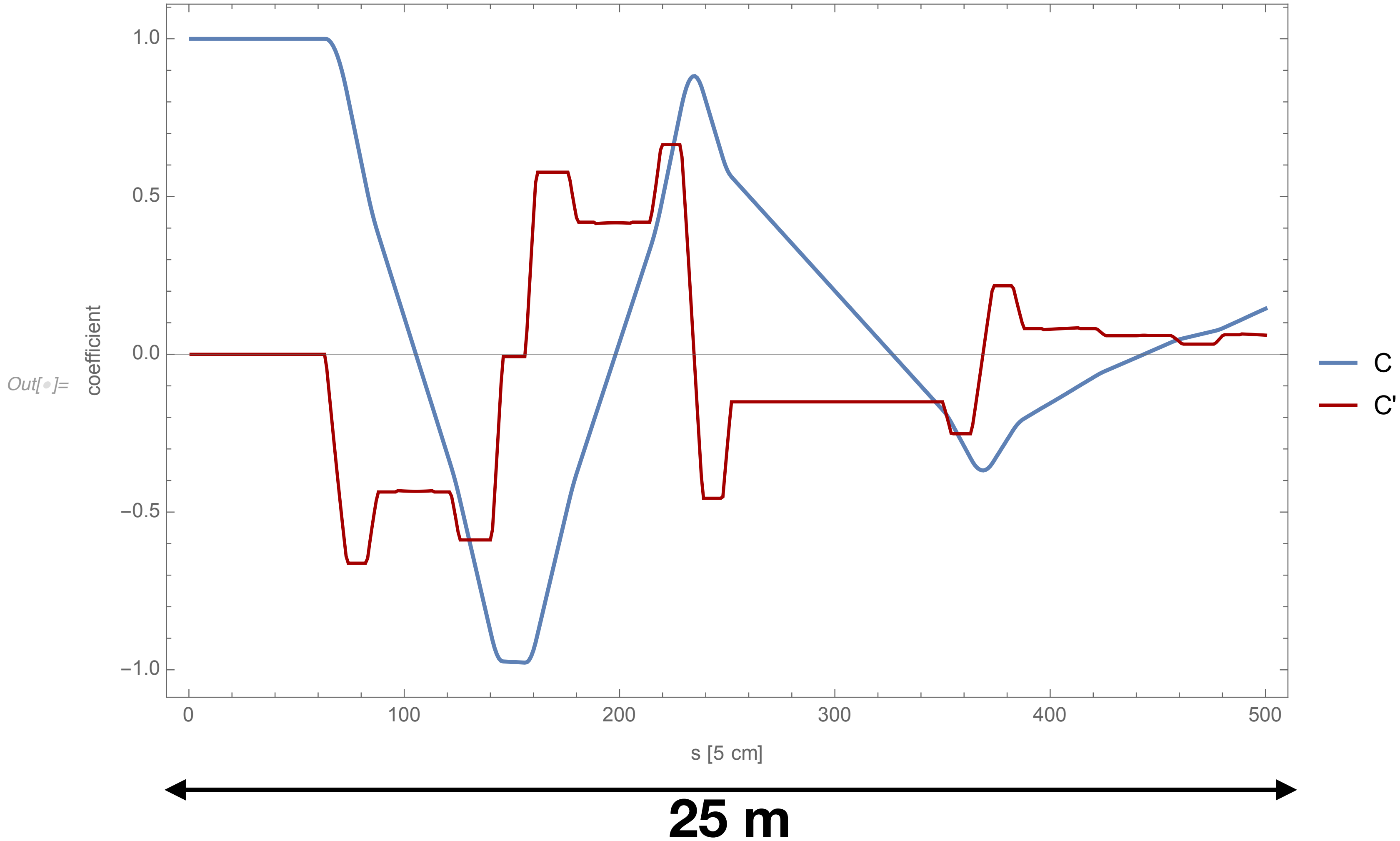


matrix elements

$$\begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}$$

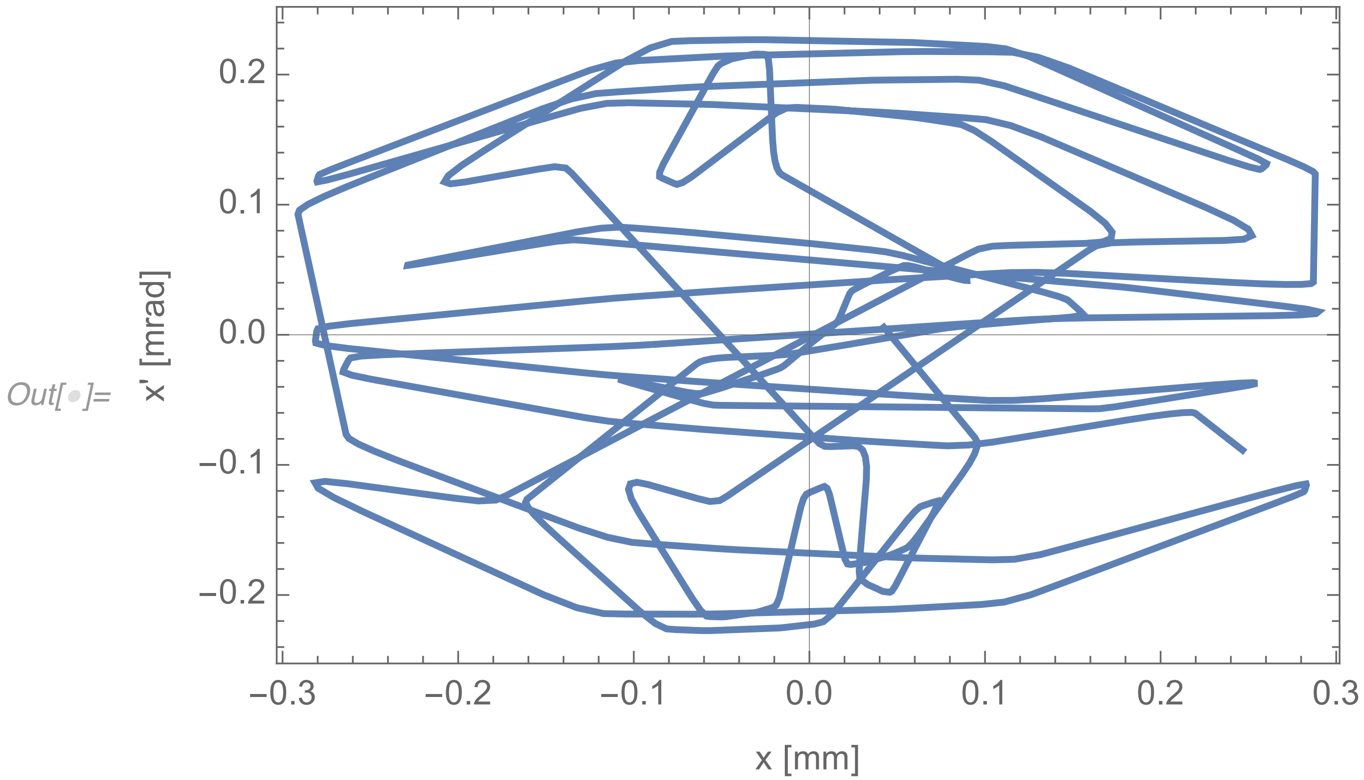
250 m





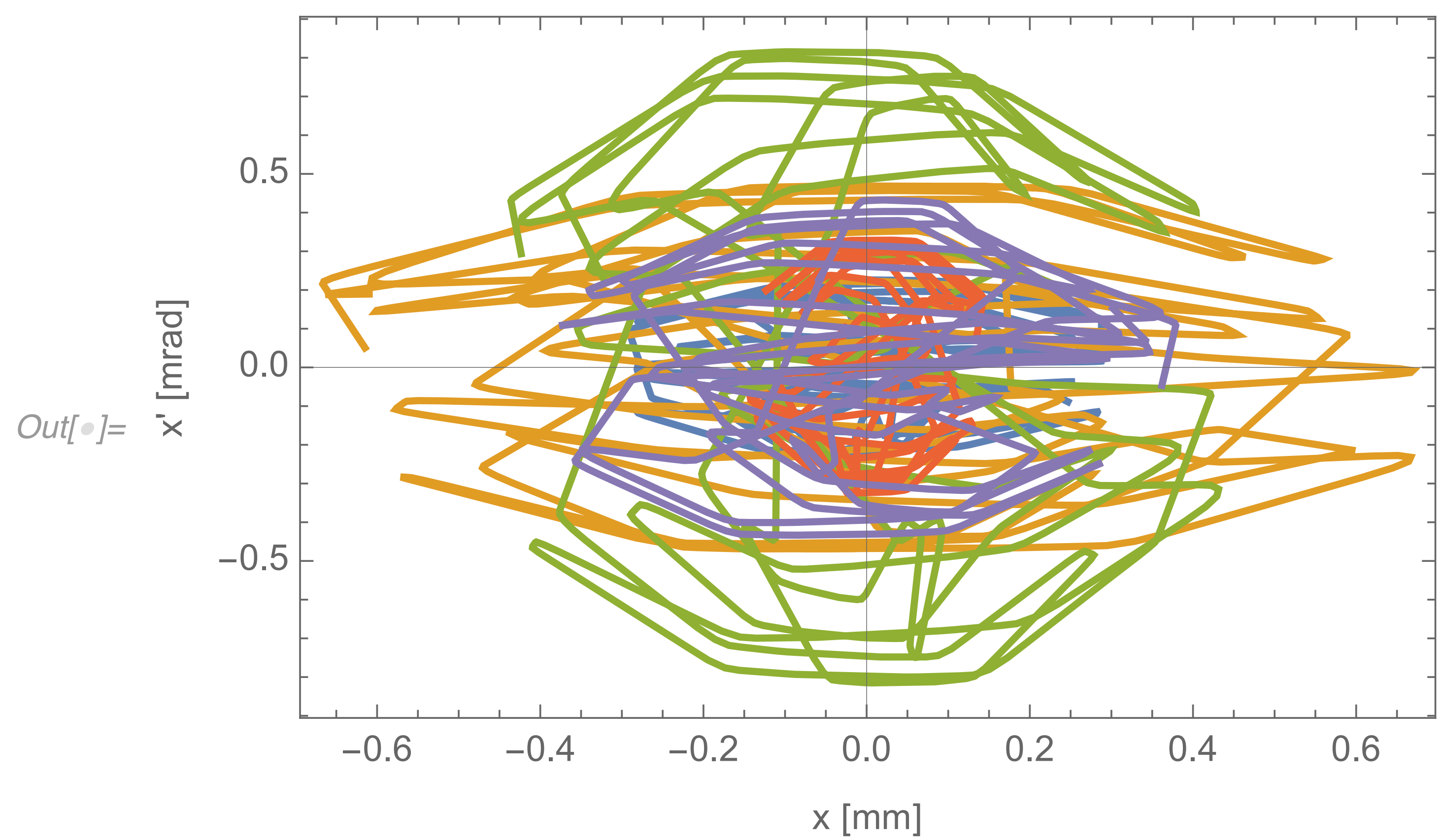
Out[•]=
coefficient
1.0
0.5
0.0
−0.5
−1.0
0
100
200
300
400
500
s [5 cm]
C
C'
25 m

trajectory in phase space



1 particle

5 particles



Important property $M = \begin{pmatrix} \cos() & 1/\sqrt{K}\sin() \\ -\sqrt{K}\sin() & \cos() \end{pmatrix}$

↳ $Det(M) = 1 = \cos^2() + \sin^2()$

since $Det(M_1) \cdot Det(M_2) = Det(M_1 \cdot M_2)$

↳ all matrices have $Det = 1$!

Or take CS-matrix

$$M_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \; ; \; Det(M_s) = CS' - SC'$$

„Wronski-det of *"

Abel : if linear d.eq. of 2. order has no first order term (yes!)

↳ $Det(M_s) = const.!$ (indep. of s)

since for $s \to 0$ $M_s = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ↝ $Det = 1$!

or ! $\frac{d}{ds}(Det(M_s)) = \frac{d}{ds}(CS' - SC')$

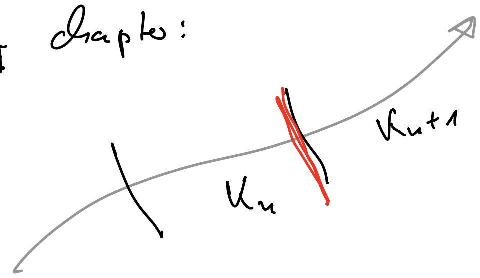
$$= C'S' + CS'' - S'C' - SC'' = CS'' - SC''$$

but ! C, S solutions of * !

$$C'' = -kC \qquad S'' = -kS$$

$$\hookrightarrow \quad CS'' - SC'' = -kCS + SkC \;\; \underline{\underline{= 0}}\,!$$

One *ugly* chapter:



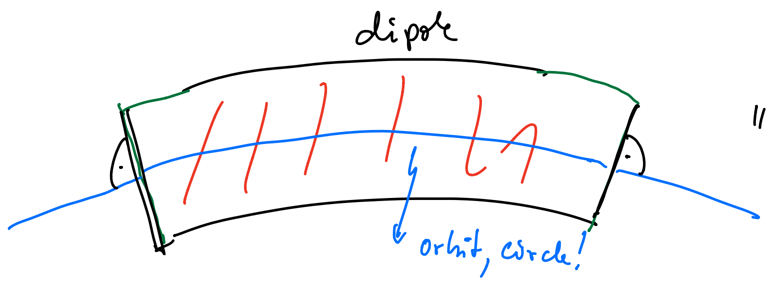
assumption: the *change* of $k_n \to k_{n+1}$ does not create effects on $\begin{pmatrix} u \\ u' \end{pmatrix}$ at the border!

1. order, linear in u!

quadrupoles: ok (2. order exists..)

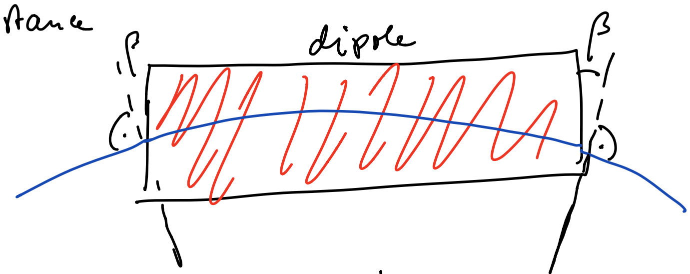
dipoles: ↯ 

problem: *if* orbit not ⊥ to dipole end (boundary)



"sector dipole"!

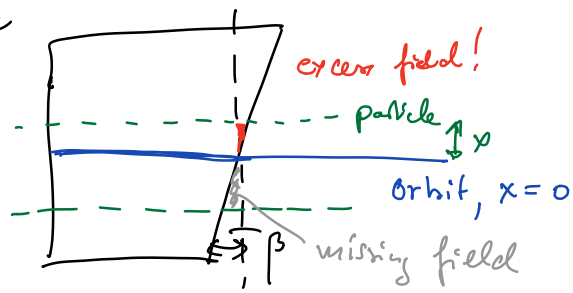
for instance



rectangular dipole

Creates first order effects!

- bending plane



add. path length inside: $\delta s = x \cdot \tan(\beta)$

add. bending $\delta\theta = \delta x' = \frac{x \cdot \tan(\beta)}{\rho_0}$ ← bending radius of dipole

„extra kick" from edge of dipole,

„thin lens" $f = \left(\frac{x}{\delta x'}\right)$

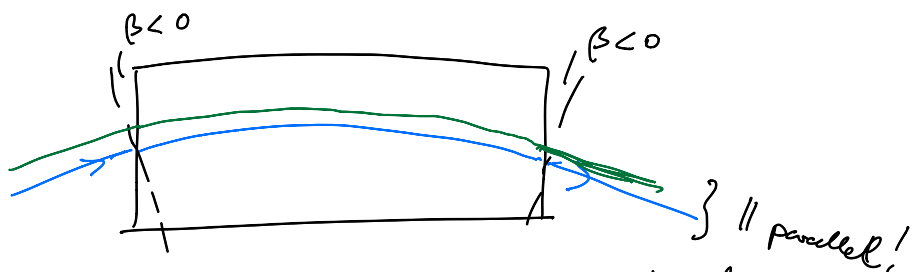
$$M_{edge,x} = \begin{pmatrix} 1 & 0 \\ -\frac{\tan\beta}{\rho_0} & 1 \end{pmatrix}$$ „edge matrix"

y-plane? (⊥ bending) more tricky, see script ....

$$M_{edge,y} \approx \begin{pmatrix} 1 & 0 \\ +\frac{\tan\beta}{\rho_0} & 1 \end{pmatrix}$$

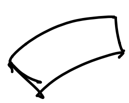
"edge focussing" or "edge defocusing", sign of β

symmetric rect. magnet.



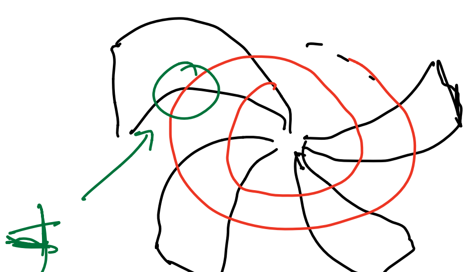
no focussing in x!

but no focussing in y!



sector: focussing in x
no-focus in y.

Application: isochronous cyclotron!
funny pole-shapes



Key

B increasing with r

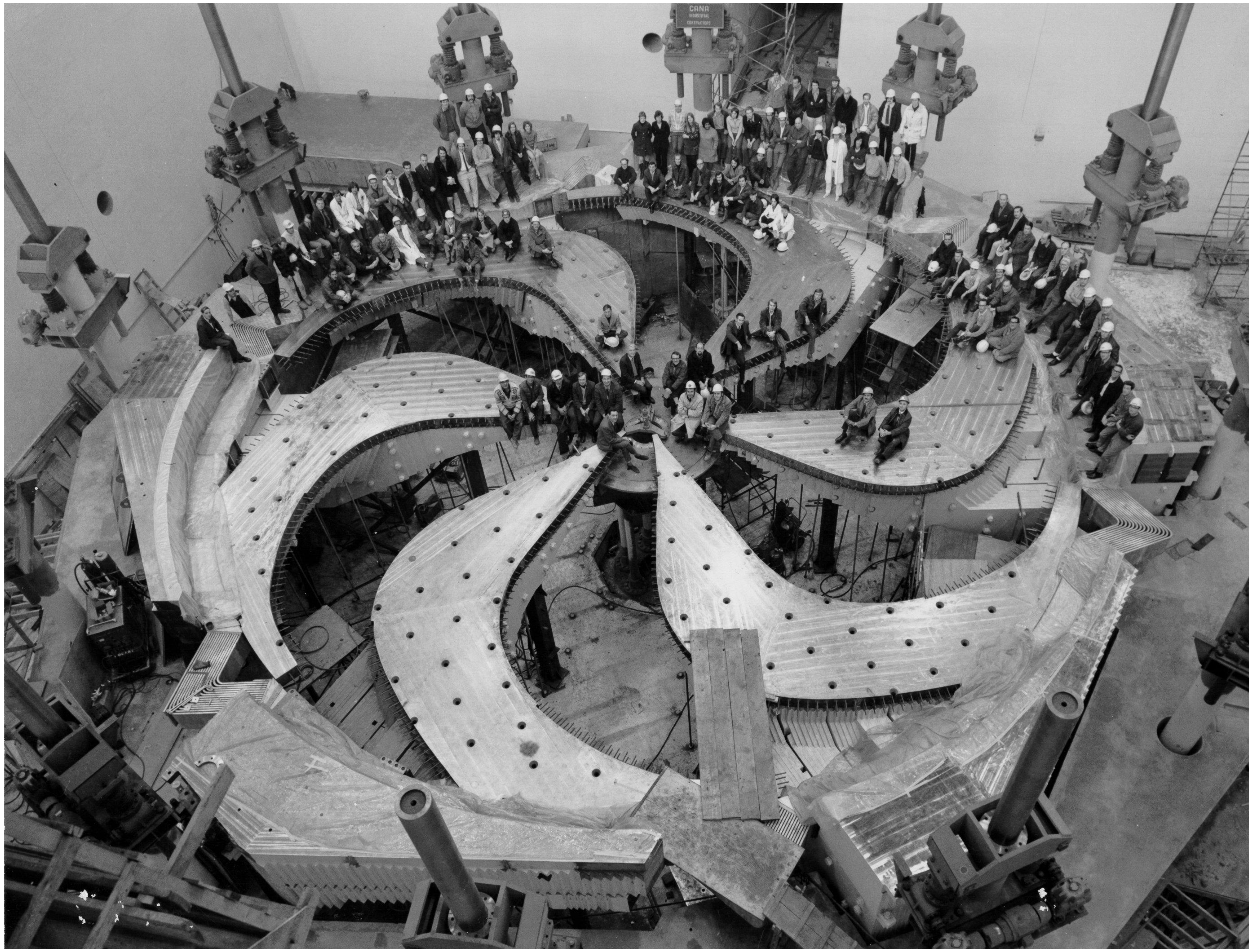
↳ vertical defoc!

Compensate by "edge focussing" of dipoles.

Isochronous Cyclotron

ω_c=const

B *increases* with r (momentum)
vertical defocussing
compensated by edge focussing

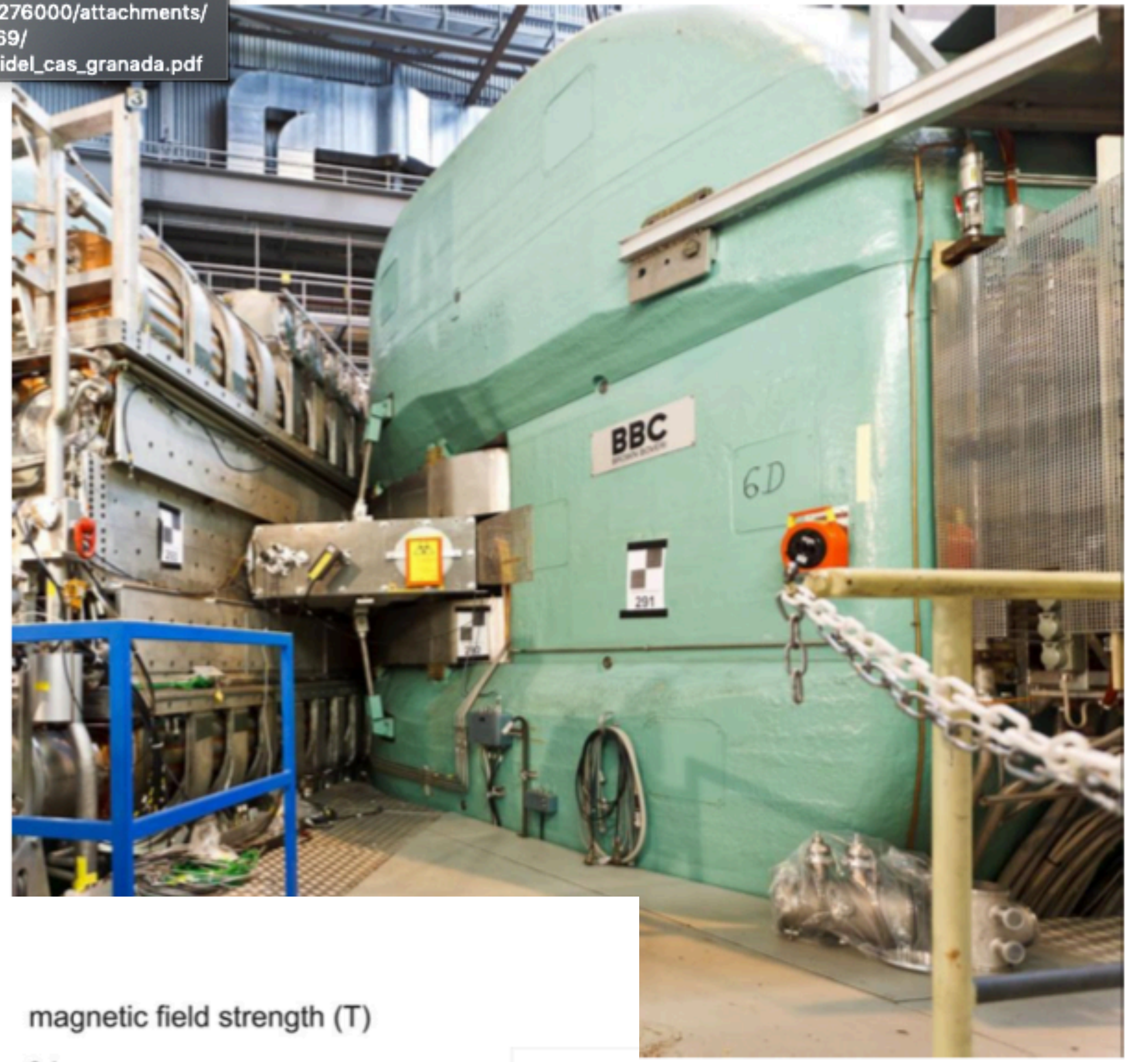


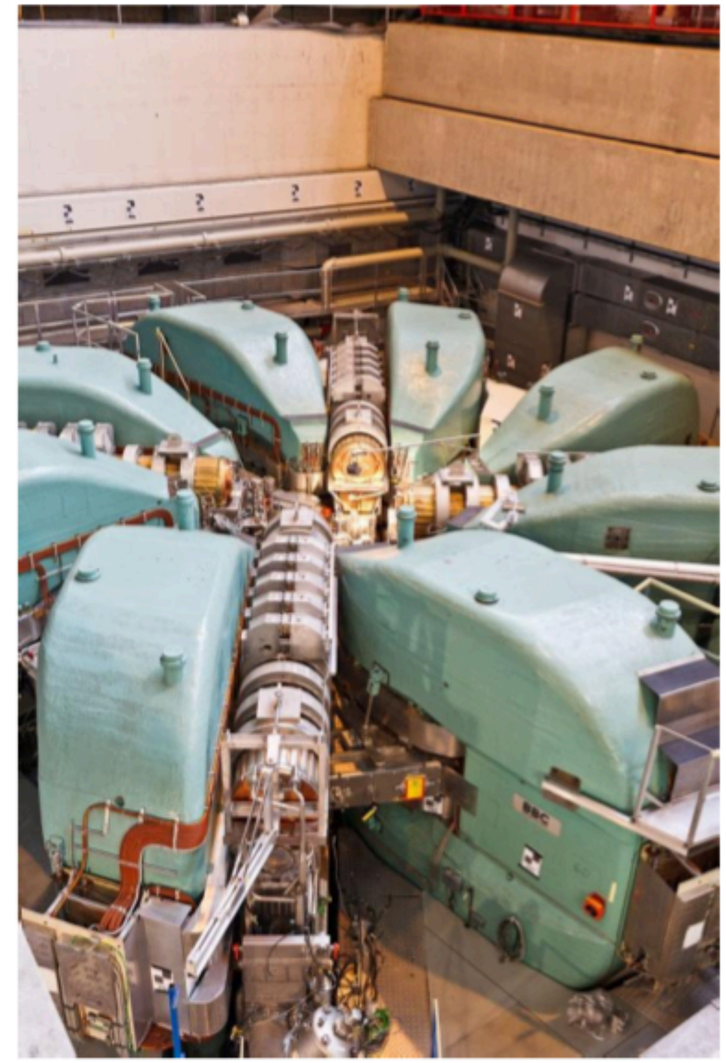
TRIUMF (Canada)

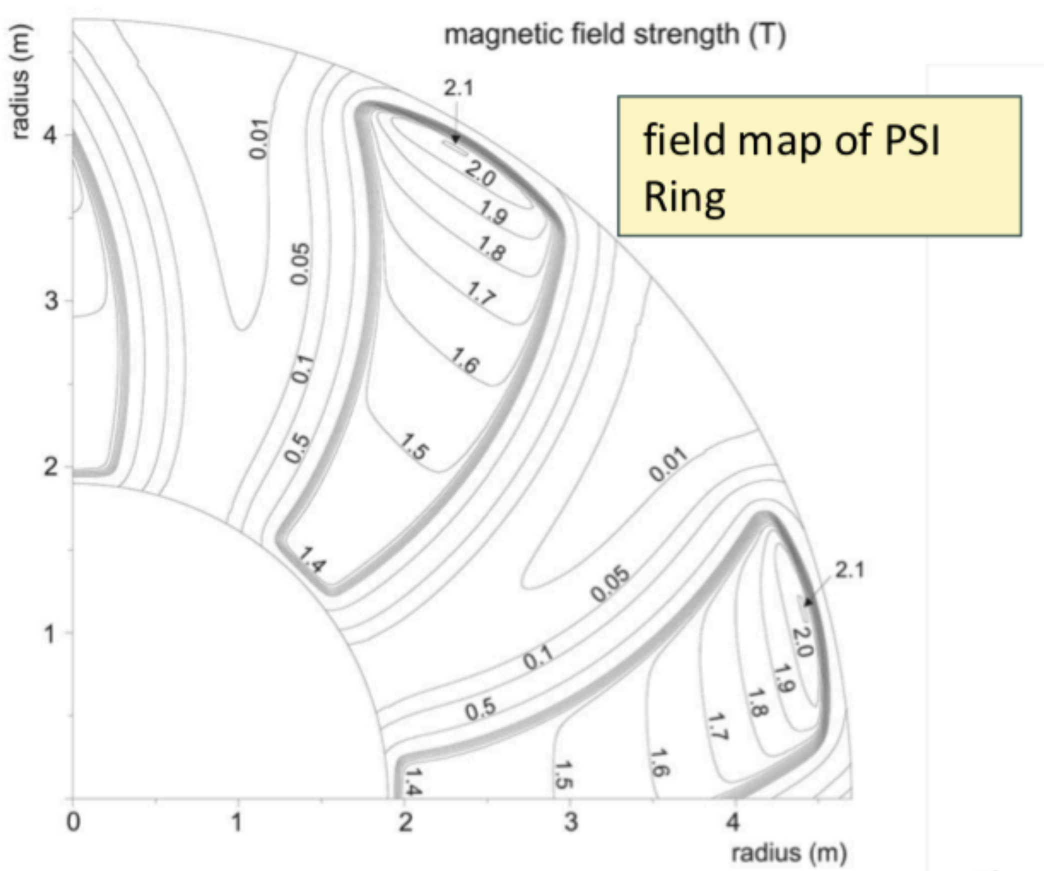
520 MeV protons
110 kW beam

picture : 1972

ico.cern.ch/event/173359/
ons/276000/attachments/
06469/
_Seidel_cas_granada.pdf







PSI (Swizerland)

595 MeV protons
1300 kW beam

Now: remember: **weak** focussing $\rho^2 \cdot k < 1$

$k < 1/\rho^2$

large machines (ρ large)

$K = \left(\frac{1}{\rho^2} - k\right)$ small! $K > 0$

but $u(s) = u_0 \cos(\,) + u_0' \frac{1}{\sqrt{K}} \sin(\,)$

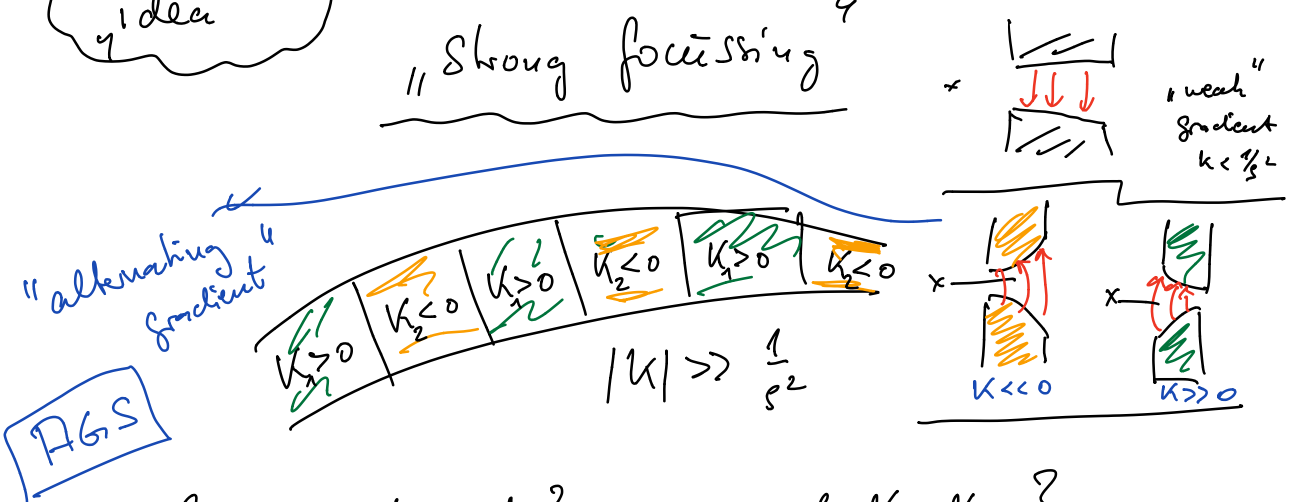
↑ finite u_0' ⇝ $u(s)$ very large!

„monster machines"

key

„idea"

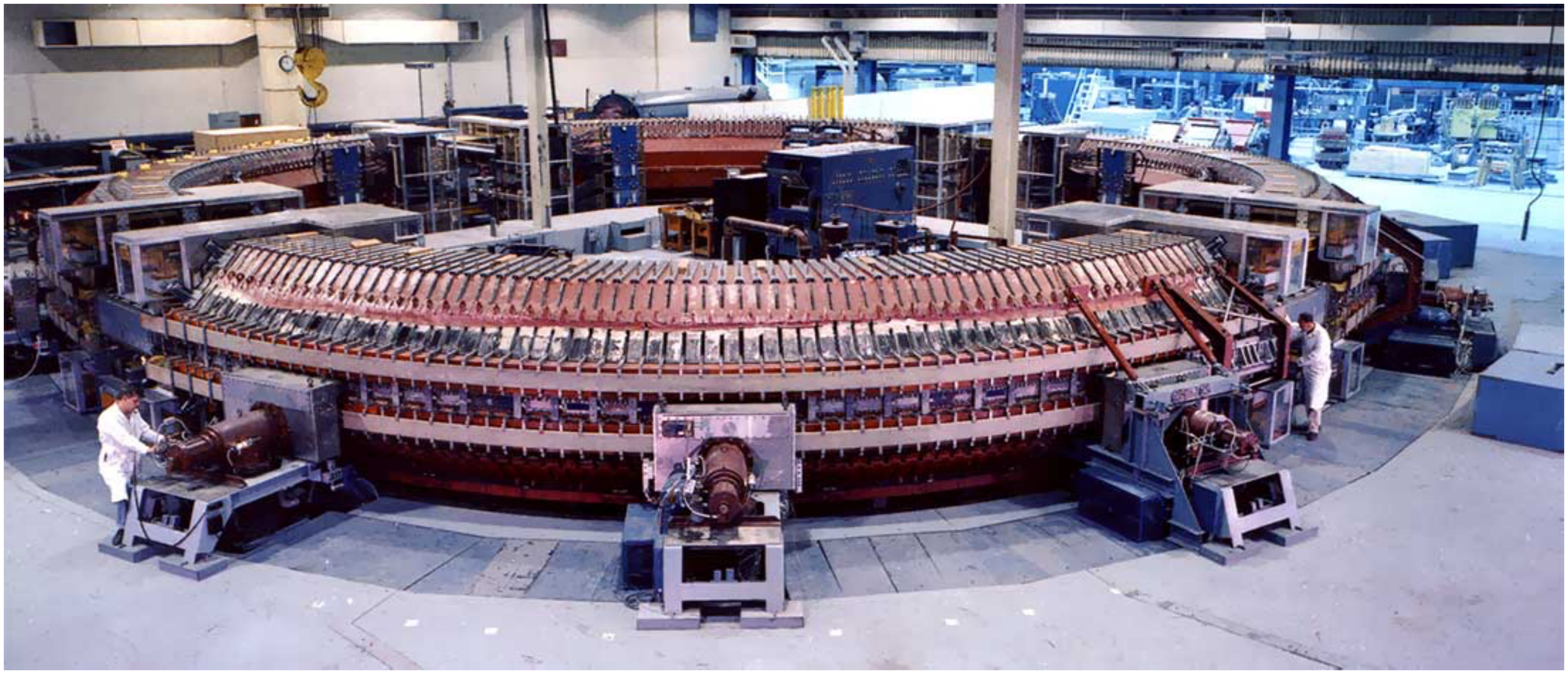
„Strong focussing"



? does it work? range of K_1, K_2?

the question of stability of circular machines (lattices).

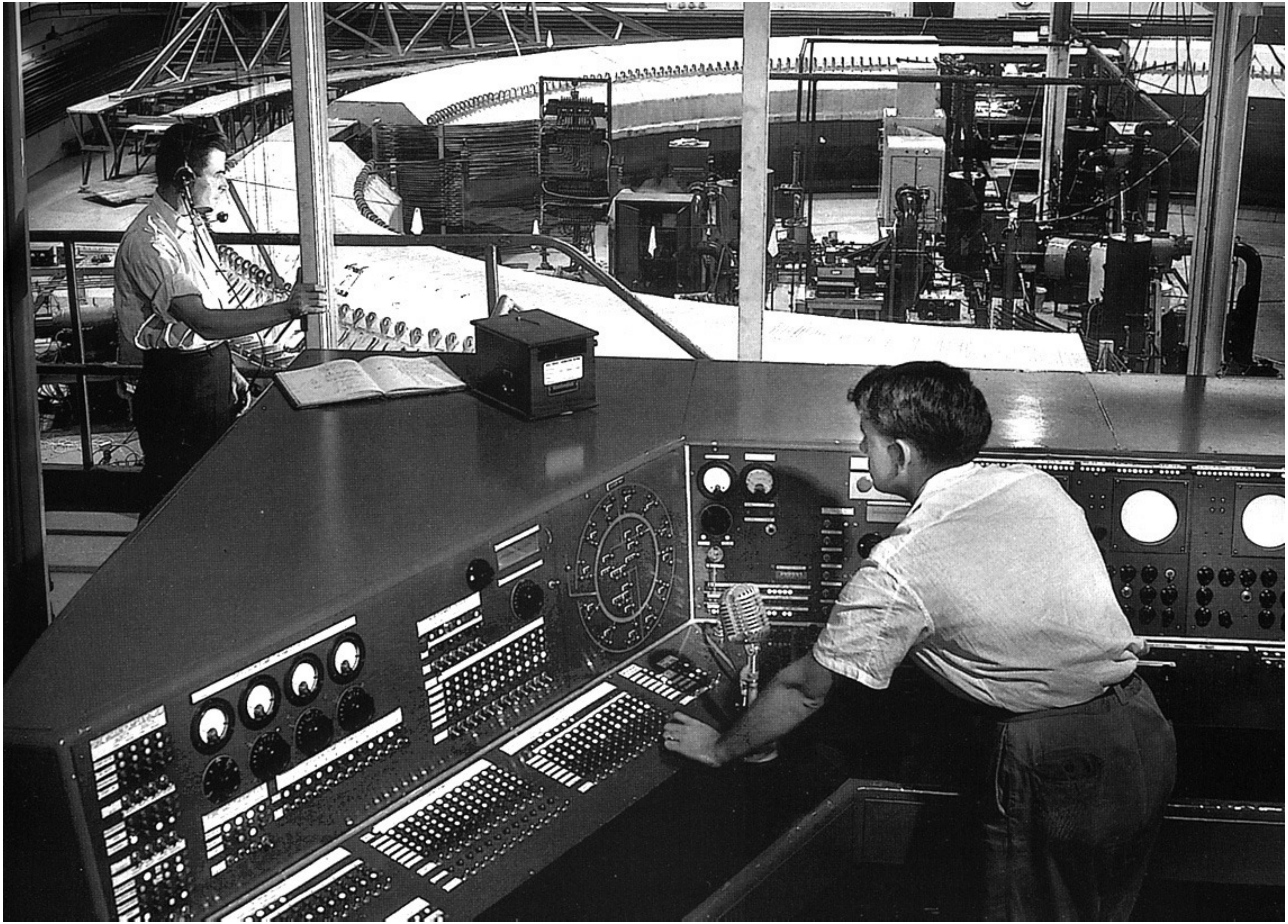
COSMOTRON (Brookhaven National Laboratory)



30 ft (9.1 m) Radius
288 1.4 T Magnete, n=0.6 2000 Tonnen
Vakuumkammer: 90 x 25 cm !
Protonen 3 GeV

1953 : 3 GeV

in operation until 1966

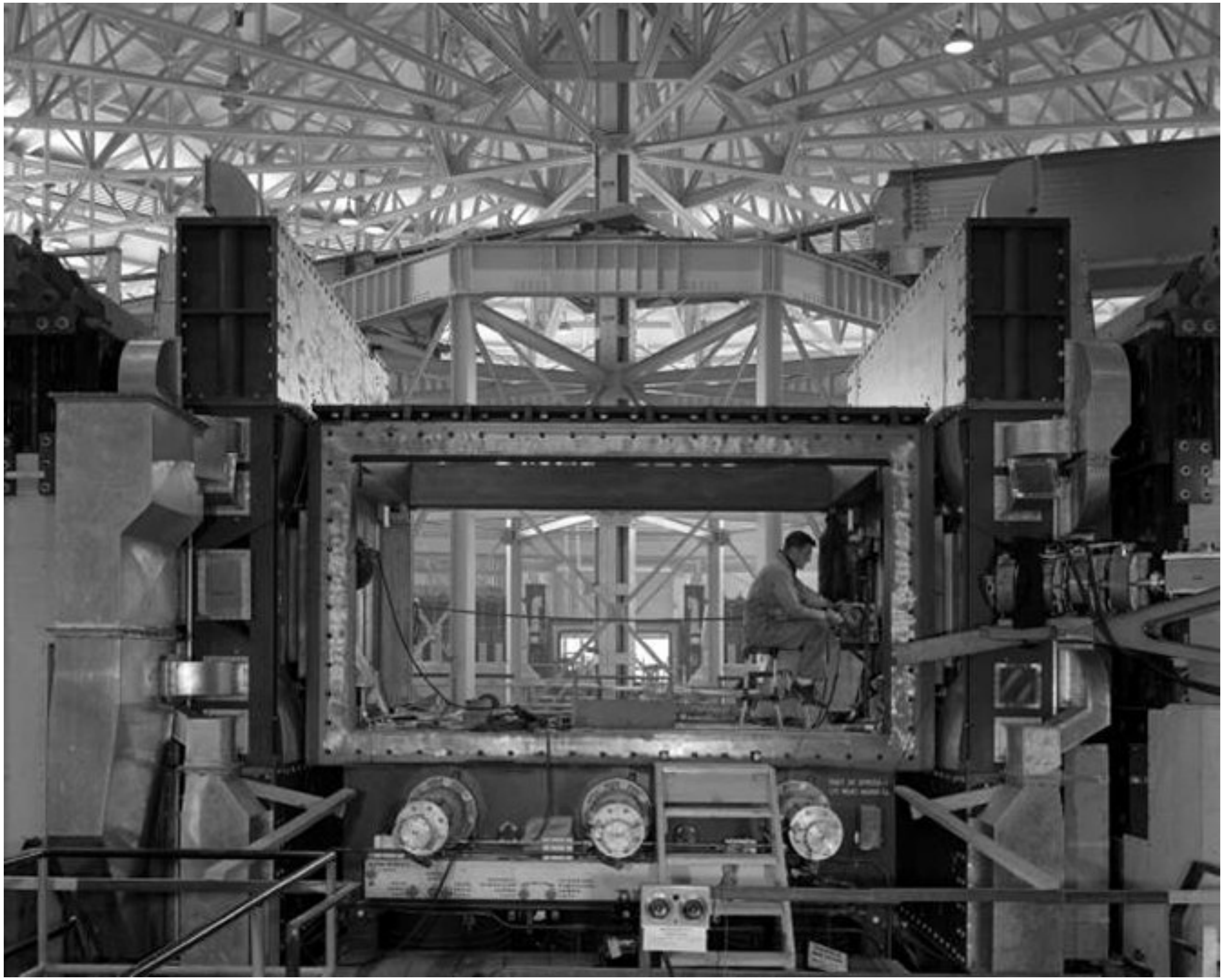


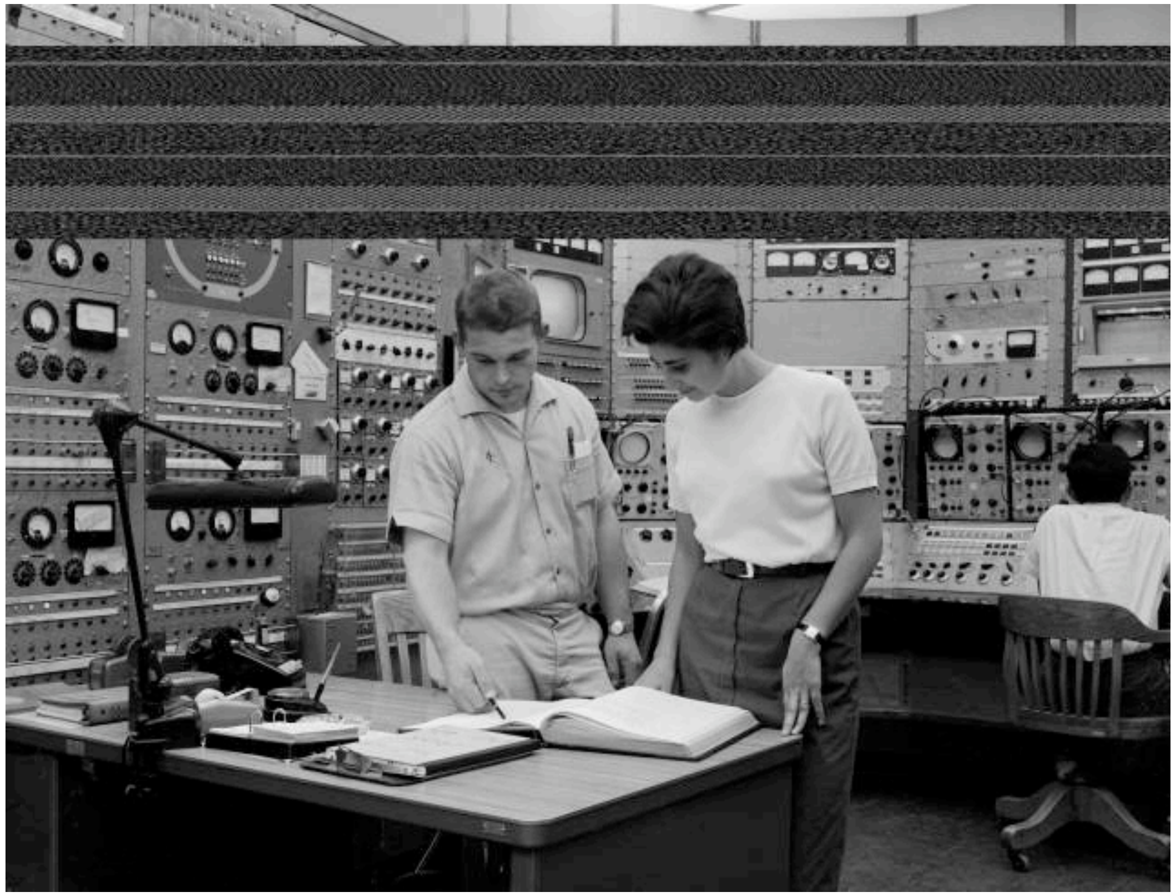
BEVATRON (Berkeley, California)

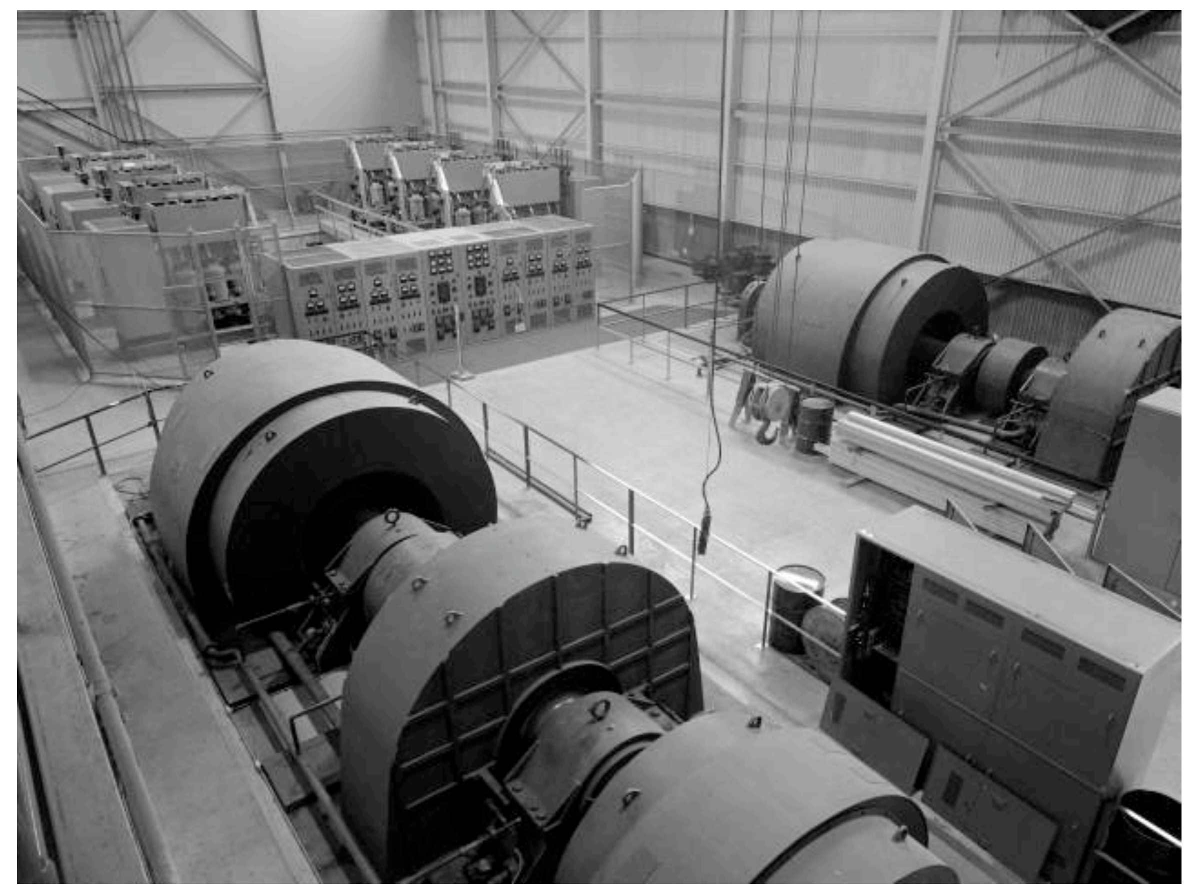
1954 : 6.2 GeV
in operation until 1993

Anti-protons !

18.2 m radius
176 1.4 T magnets 10 000 tons

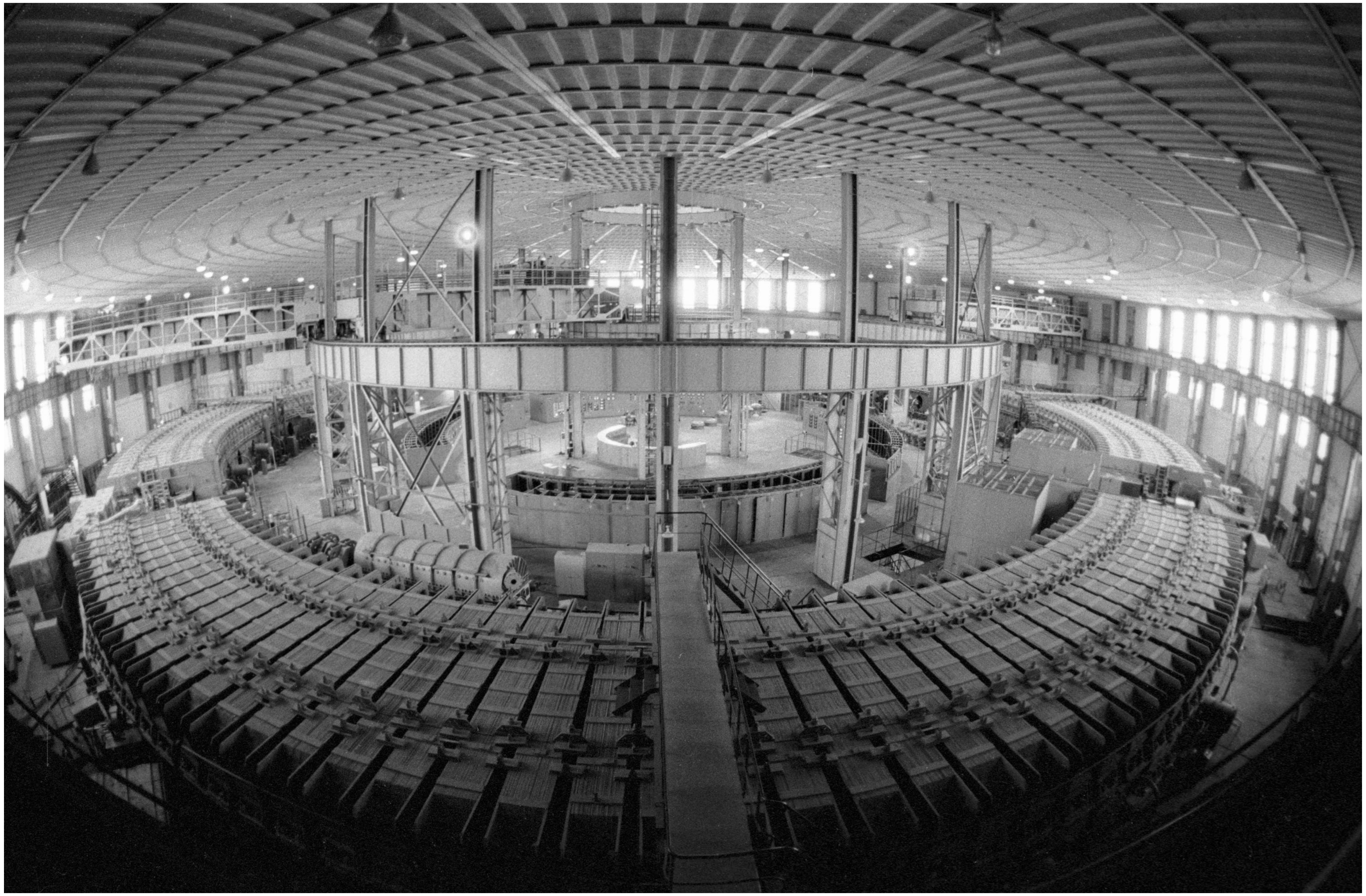






Synchrophasotron (Dubna, JINR)

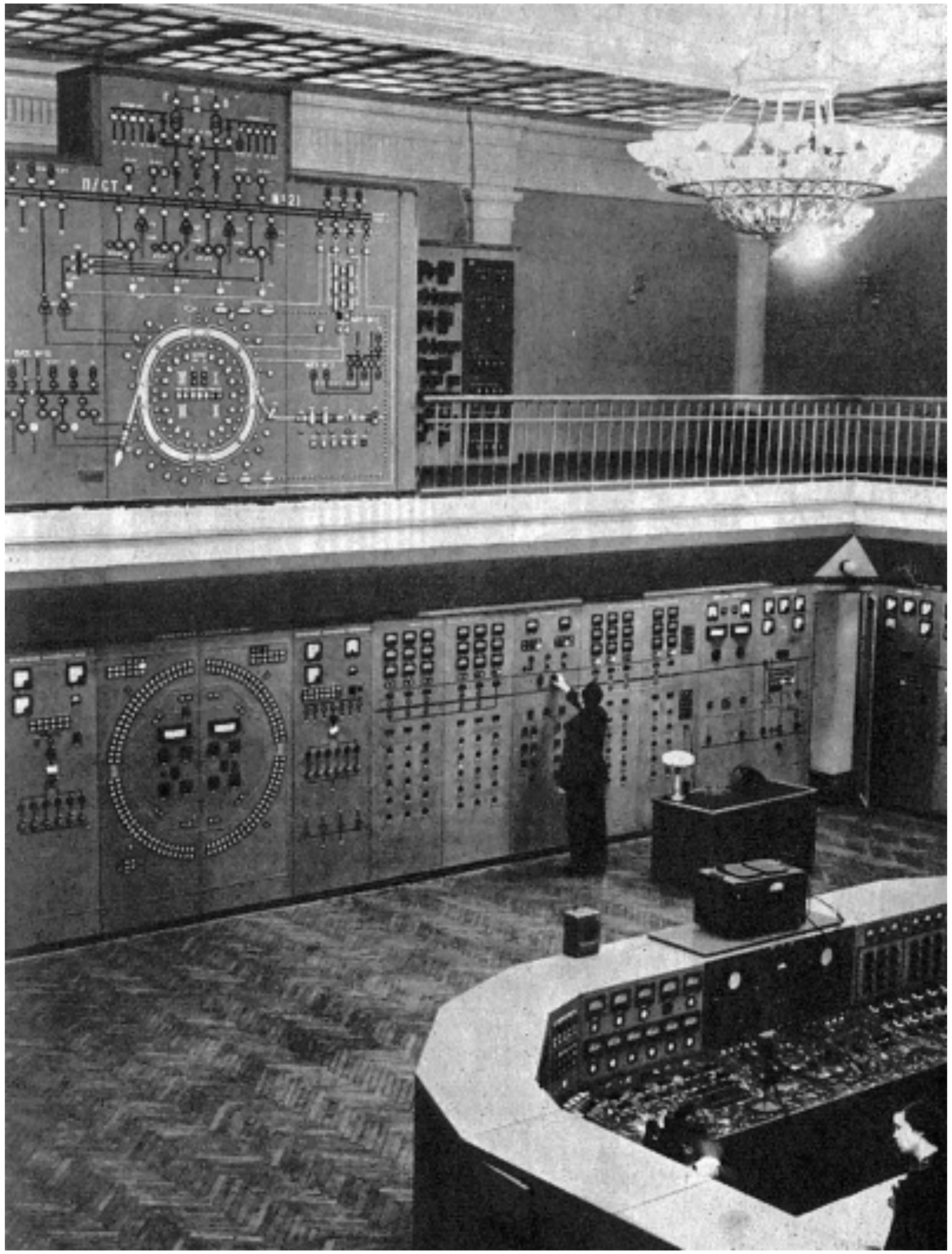
1957, 10 GeV
in operation until 2003



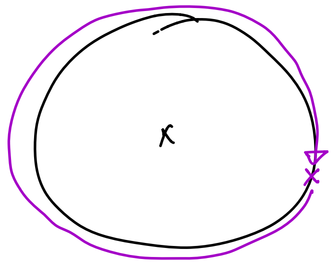
33 m radius
36 000 tons iron

scaling of radius and diameter vacuum chamber !

control room



subway station Moscow (1999)

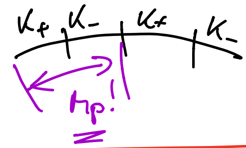


circular machine ~ periodic magnet lattice

after 1 turn ~ sequence repeats.

M_p : period matrix

(periodicity could be shorter than full turn..)



1 turn : M_p

2 " " $M_p \cdot M_p$

3 " $M_p \cdot M_p \cdot M_p$

⋮

stable? if $(M_p)^N$ = finite!

say: $M_p = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$ (four numbers!)

↳ new "form" . . .

$M_J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$

$M_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

any (2x2) matrix can be written as

$$M_p \stackrel{!}{=} M_I \cdot \cos\mu + M_J \cdot \sin\mu$$

$\alpha, \beta, \gamma, \mu$ — 4 new parameters.

$$= \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

"Twiss-form"

$(C, S, C', S') \Longleftrightarrow (\alpha, \beta, \gamma, \mu)$

I) $\quad Tr(M_p) = C + S' = \cos\mu + \alpha\sin\mu + \cos\mu - \alpha\sin\mu = 2\cos\mu$

$$\cos\mu = \frac{1}{2}(C+S') = \frac{1}{2} Tr M$$

II) $\quad S = \beta \cdot \sin\mu \quad \curvearrowright \quad \beta = \frac{S}{\sin\mu} \qquad \gamma = -\frac{C'}{\sin\mu}$

$$\alpha = \left(\frac{C - S'}{2\sin\mu}\right)$$

we know: $\quad Det(M_p) = 1 \quad \curvearrowright$

$$(\cos\mu + \alpha\sin\mu)(\cos\mu - \alpha\sin\mu) = \cos^2\mu - \alpha^2\sin^2\mu$$

$$Det(M) = \cos^2\mu - \alpha^2\sin^2\mu + \beta\gamma\sin^2\mu \overset{!}{=} 1$$

$$\Rightarrow \boxed{\beta\gamma - \alpha^2 \overset{!}{=} 1} \qquad \gamma = \frac{1+\alpha^2}{\beta}$$

$(M_p)^N$?

$\boxed{M}$

$$(M_p)^N = \begin{pmatrix} \cos(N\mu) + \alpha \sin(N\mu) & \beta \sin(N\mu) \\ -\gamma \sin(N\mu) & \cos(N\mu) - \alpha \sin(N\mu) \end{pmatrix} \qquad N \in \mathbb{N}$$

finite if μ real! $\Longleftrightarrow$ $|\cos\mu| < 1$

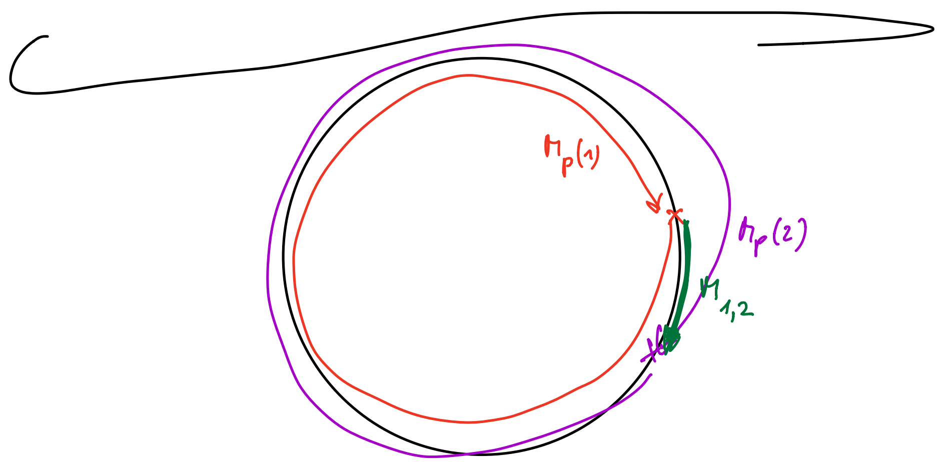
$2\,|\cos\mu| < 2$

$\mathrm{Tr}(M_p) < 2$!

very simple check!

add two numbers $(C+S') < 2$ ok

≥ 2 $\Rightarrow$ unstable



$M_p(1) \neq M_p(2)$! $\qquad \beta_1, \alpha_1, \gamma_1 \neq \beta_2, \alpha_2, \gamma_2$

but $\mu_1 = \mu_2$!

proof: "similarity transf."

$$M_2 = \left(M_{1,2} \cdot M_1 \cdot (M_{1,2})^{-1} \right)$$

$\mathrm{Tr}(M)$ invariant under sim. transf.

- μ : universal parameter of the (periodic) lattice
- β, α, γ : unique functions of the starting point s of μ_p

β-functions — the holy grale of (linear) optics

(Twiss parameters / " functions / Optical functions ⋮)

Is „strong focussing" (alternating gradients) possible? (stable) → Yes, if ....

revolution of the 50'th., basis for all (large) modern machines.

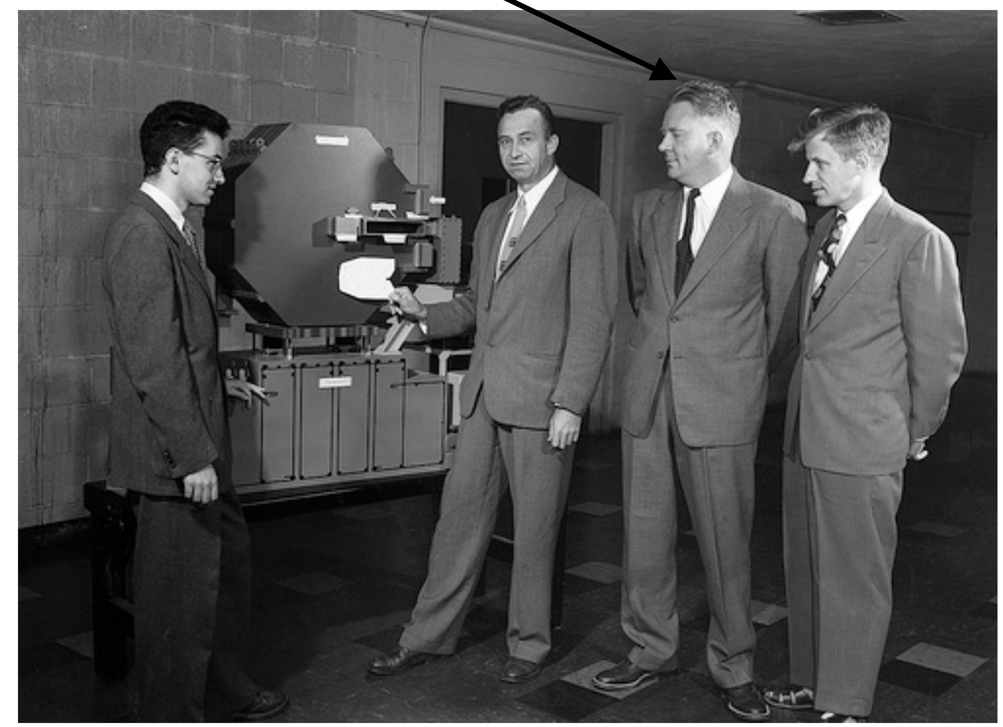
key history.

E.D.Courant
*1920 in Göttingen

M.S. Livingston
1905-1986

H.S. Snyder 1913-1963

Ernest D. Courant



PHYSICAL REVIEW VOLUME 88, NUMBER 5 DECEMBER 1, 1952

The Strong-Focusing Synchroton—A New High Energy Accelerator*

ERNEST D. COURANT, M. STANLEY LIVINGSTON,† AND HARTLAND S. SNYDER
Brookhaven National Laboratory, Upton, New York

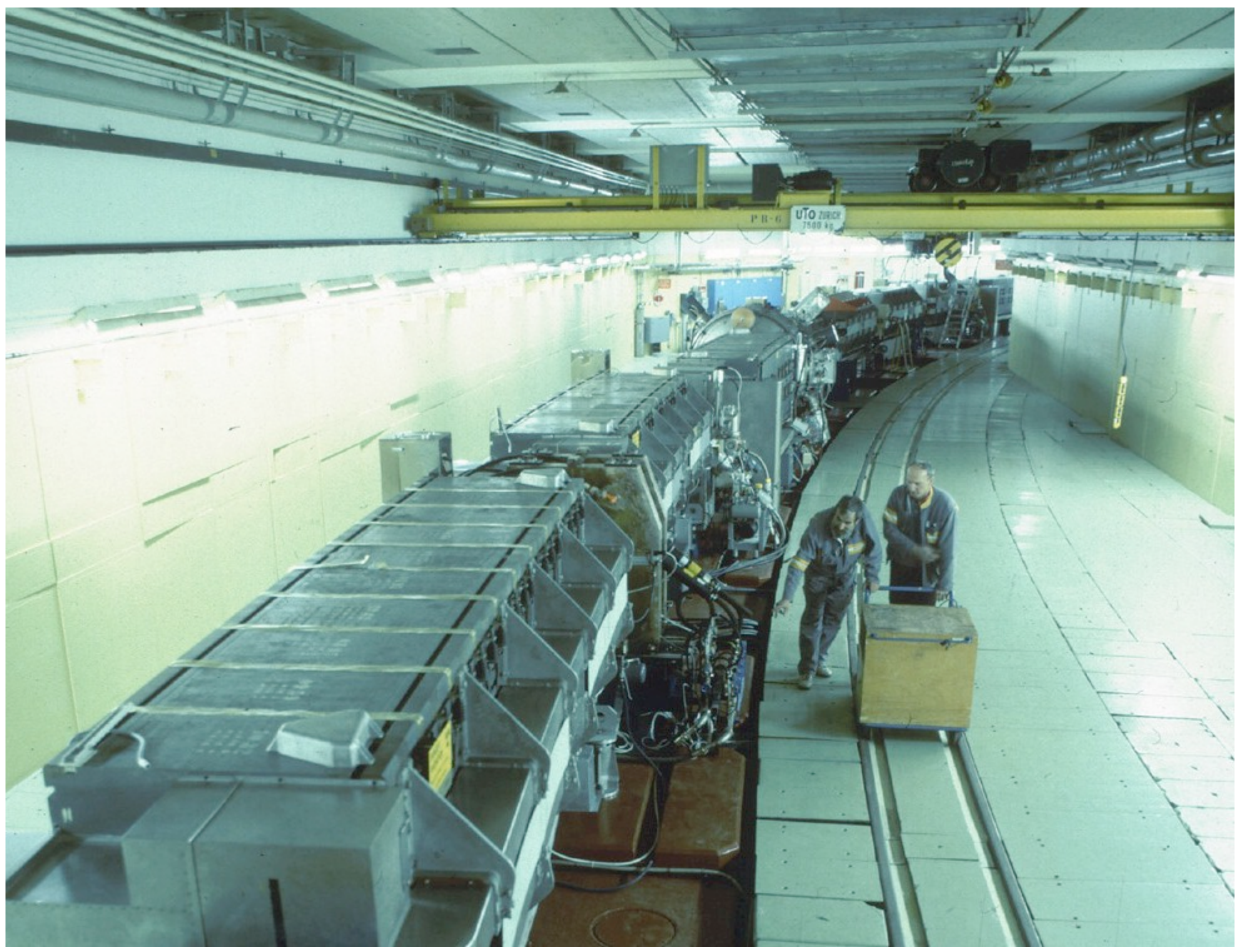
(Received August 21, 1952)

Strong focusing forces result from the alternation of large positive and negative *n*-values in successive sectors of the magnetic guide field in a synchrotron. This sequence of alternately converging and diverging magnetic lenses of equal strength is itself converging, and leads to significant reductions in oscillation amplitude, both for radial and axial displacements. The mechanism of phase-stable synchronous acceleration still applies, with a large reduction in the amplitude of the associated radial synchronous oscillations. To illustrate, a design is proposed for a 30-Bev proton accelerator with an orbit radius of 300 ft, and with a small magnet having an aperture of 1×2 inches. Tolerances on nearly all design parameters are less critical than for the equivalent uniform-*n* machine. A generalization of this focusing principle leads to small, efficient focusing magnets for ion and electron beams. Relations for the focal length of a double-focusing magnet are presented, from which the design parameters for such linear systems can be determined.

„the“ paper (1952)

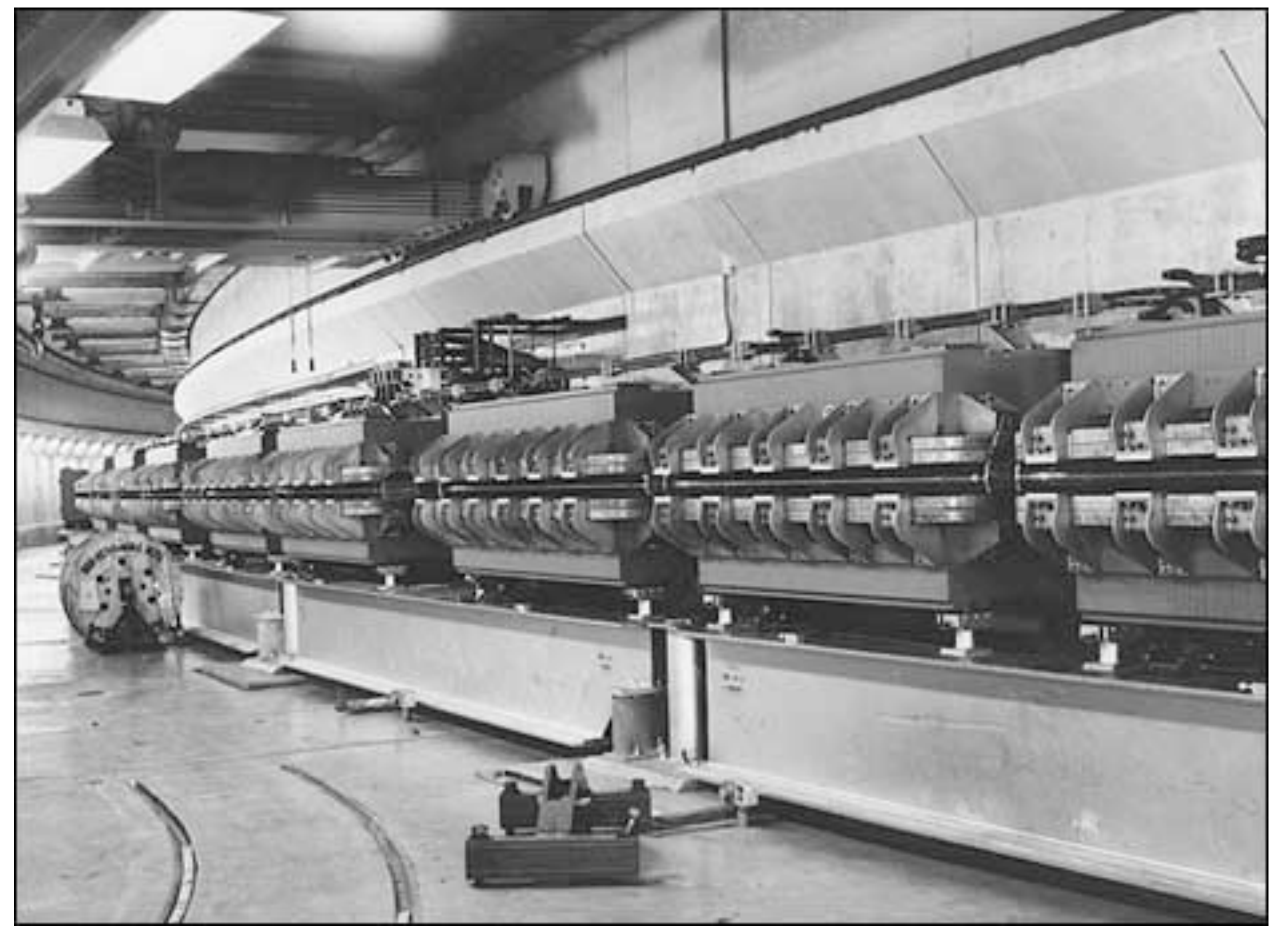
CERN -PS

28 GeV , since 1959 in operation



Brookhaven „AGS“

33 GeV , since 1960 in operation



Elektron „AGS“ machines

Cornell University 1.2 GeV (1954)

...

Univ. Bonn, 500 MeV (since 1958)

....

DESY, 6 GeV (since 1964)

....

CERN, LEP up to 104 GeV (1989-2000)

Nicholas Christofilos (1916 - 1972)

United States Patent Office

2,736,799

Patented Feb. 28, 1956

1

2,736,799

FOCUSSING SYSTEM FOR IONS AND ELECTRONS

Nicholas Christofilos (or Philos), Athens, Greece

Application March 10, 1950, Serial No. 148,920

8 Claims. (Cl. 250—27)

The present invention relates to a new focussing system for ions and electrons and application thereof in particle accelerators.

A major problem in the design of particle accelerators is the provision of suitable means for focussing the accelerated particles towards a predetermined orbit and compensating the mutual electrostatic repulsive forces.

An ideal focussing system must accelerate the moving particles towards a predetermined orbit from all directions and the focussing forces must increase as the distance from said orbit increases.

If we consider an orthogonal coordinate system, $x, y, z,$ and suppose that the particle's orbit coincides with the x-axis and considering as P_x, P_y, P_z the $x, y, z,$ components of the focussing forces, then, in an ideal focussing system the equations of the P_x, P_y, P_z would be

$$P_x=0 \quad (1)$$

$$P_y=-\epsilon_y \cdot y \quad (1a)$$

$$P_z=-\epsilon_z \cdot z \quad (1b)$$

But simultaneously the Laplace equation $\Delta\varphi=0$ must be satisfied so that it must be

$$\frac{\delta P_x}{\delta x}+\frac{\delta P_y}{\delta y}+\frac{\delta P_z}{\delta z}=0 \quad (1c)$$

or

$$\epsilon_y=-\epsilon_z \quad (1d)$$

From the above equations is shown that a focussing field capable to accelerate ions or electrons towards a predetermined orbit from all directions simultaneously is impossible. Therefore the focussing system proposed herein is based in a new principle, namely:

If, along a predetermined orbit of ions or electrons an electrostatic or electromagnetic field is produced by means of suitably arranged conductors (connected to a high voltage source or energized by high intensity current) exerting on the moving, along said orbit, particles (ions or electrons) forces directed normally to said orbit and varying periodically, in direction and magnitude along said orbit, and increasing in magnitude as the distance from said orbit increases, then the mean value of the focussing forces is negative (directed towards the orbit) and the particles are focussed towards the orbit from all directions.

The focusing forces acting on the particles resulting from the field which is produced electrostatically or electromagnetically, increase as the distance from the orbit increases. The particles move at some finite distance from the orbit and in a direction substantially parallel to the orbit by virtue of the periodically varying exciting focusing forces due to the field. The particles undergo forced oscillations and are subject to the alternately converging and diverging forces from the field. The electrically produced force field, electromagnetic or electrostatic, imposed upon the orbit and the path of the particles exerts forces on the particles within a plane whose normal is substantially parallel to the velocity vector of each of the particles. The path of the particles becomes concave towards the orbit in a converging section and convex towards the orbit in the diverging section. Since the forces are greater as the distance from the orbit becomes greater, the mean value of the converging and diverging forces along the converging section is greater than the mean value of the forces along the diverging section. The resultant force and net effect of the mean value of these alternating forces causes the particles in the path to be forced towards the orbit from all directions and focusing is thereby obtained.

2

In a focussing system based upon this principle the $x, y, z,$ components of the focussing forces are

$$P_x=0 \quad (2)$$

$$P_y=-\epsilon \cdot y \cdot \sin\frac{2\pi x}{\lambda} \quad (2a)$$

$$P_z=\epsilon \cdot z \cdot \sin\frac{2\pi x}{\lambda} \quad (2b)$$

As it is obvious

$$\Delta\varphi=\frac{\delta P_x}{\delta x}+\frac{\delta P_y}{\delta y}+\frac{\delta P_z}{\delta z}=0$$

$$\epsilon_y=-\epsilon_z \quad (1d)$$

so that the Laplace equation is satisfied and therefore the production of such a field is possible. If we consider a particle moving parallel to the x-axis and at a distance $z=z_0$, $y=0$ the force exerted on said particle is

$$P_z=e \cdot \epsilon \cdot z_0 \sin \cdot \frac{2\pi x}{\lambda} \quad (3)$$

As the force P_z varies periodically as the particle moves along the x-axis, said particle undergoes forced oscillations of frequency

$$f=\frac{\beta c}{\lambda} \quad (4)$$

where βc the velocity of the particle.

The result of these oscillations is that the distance from the orbit oscillates around the mean value z_0 according to the equation

$$z=z_0\left(1-\mu \sin\frac{2\pi x}{\lambda}\right) \quad (5)$$

where

$$0<\mu<1 \quad (6)$$

In the region where

$$\sin\frac{2\pi x}{\lambda}$$

is negative the mean value of the distance from the orbit is greater than z_0 while in the region where

$$\sin\frac{2\pi x}{\lambda}$$

is positive the mean value of the distance is less than z_0, so that the mean value of the force in the first region is greater than the mean value in the second region, with the result that the mean value of the force in a length λ is negative, focussing the particle towards the x-axis, from all directions.

If the maximum value of the force is

$$P_{max}=e \cdot \epsilon \cdot z_0 \quad (7)$$

then the mean value P_m is

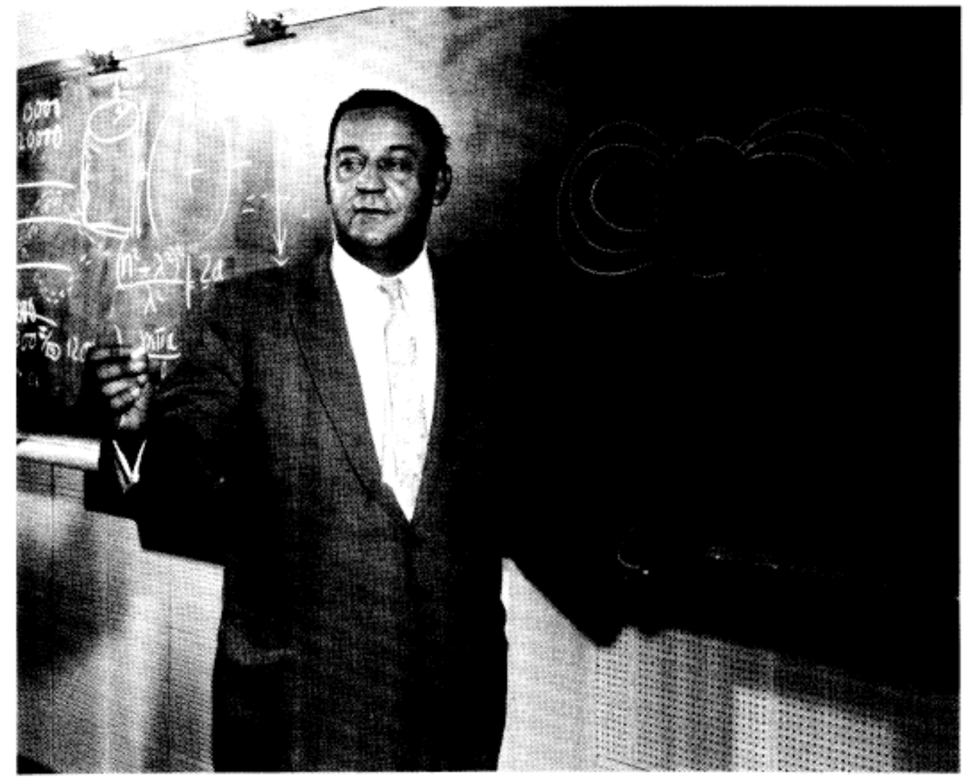
$$P_m=e \cdot \epsilon_m \cdot z_0 \quad (8)$$

where

$$\epsilon_m=\epsilon \cdot \frac{\mu}{2} \quad (9)$$

and

$$\mu=\frac{e\lambda^2}{4\pi^2\beta^2 V} \quad (10)$$



no publication, US Patent submitted 1950
accepted 1956

principle described in detail, example calculated for a 6GeV Synchrotron with K = ±250

the story behind the CERN-PS

The PS group set out to work on a synchrotron of the **weak-focusing type, similar to the Cosmotron** at Brookhaven, but with an energy of 10 GeV. Three members of the group, **Dahl, Goward and Wideröe**, went to Brookhaven in August 1952 for discussions with the Cosmotron's designers. The American scientists, however, presented their visitors with a **revolutionary concept for the design of future high-energy accelerators. Alternating the magnetic-field gradients**

Odd Dahl

Dahl hatte keine universitäre Ausbildung, sondern bildete sich als Elektriker und Radiotechniker aus. Er erkundete mit einem Elektroingenieur die Möglichkeit per Radio mit Fischerbooten zu kommunizieren, was zu seiner ersten Publikation führte. 1921 nahm er Flugstunden.
1922 nahm er als Pilot an der „Maud"- Expedition von Roald Amundsen zum Erkunden der Nordostpassage teil. Da das Flugzeug beim Versuch des Abhebens vom Eis beschädigt wurde, verbrachte er statt mit Flugerkundungen zwei Jahre mit geophysikalischen Beobachtungen in der sibirischen Arktis (zusammen mit dem Geophysiker Harald Ulrik Sverdrup), wobei er Zeit fand, sich unter Anleitung von Sverdrup mit Physik zu befassen und ein geschickter Instrumentenbauer wurde.
Nach der Rückkehr wurde er Assistent von Sverdrup und ging 1926 an die Carnegie Institution for Science in Washington, D.C., wo er mit Merle Antony Tuve und Lawrence Hafstad einen der ersten Van-de-Graaff-Generatoren baute. Außerdem baute er Instrumente zum Studium des Erdmagnetfelds und der Kennelly-Heaviside-Schicht. Ab 1936 war er am Christian Michelsen Institut (CMI) in Bergen, wo er drei Van de Graaff Generatoren und ein Betatron baute.
Nach dem Zweiten Weltkrieg leitete er den Bau eines norwegisch-niederländischen experimentellen Kernreaktors in Norwegen (mit Gunnar Randers vom Institut für Atomenergie in Norwegen und Ingenieuren des CMI), dem ersten außerhalb der eigentlichen Nuklearmächte.
1951 wurde er von Pierre Auger und Edoardo Amaldi eingeladen, an der Projektierung des CERN mitzuarbeiten. Ab 1952 leitete er das Proton-Synchrotron Projekt des CERN, das zum PS führte. Zuerst sollte die Maschine ähnlich dem Cosmotron werden, nur mit höherer Leistung (10 bis 15 GeV). Bei einem Besuch (mit Frank Goward und Rolf Wideröe) 1952 am Brookhaven National Laboratory, das den Cosmotron betrieb, erfuhren sie vom neuen Starken-Fokussierungskonzept von M. Stanley Livingston, Ernest Courant und Snyder, das viel höhere Energien zu erreichen versprach. Dahl richtete das Projekt sofort danach aus (er wies die CERN Mitarbeiter an *alles stehen und liegen zu lassen und nur noch an dieser Aufgabe zu arbeiten*[1]) und überzeugte die CERN Führung davon. Die Arbeit daran erforderte statt Ingenieuraufgaben neue Forschungen, und die Leitung der Entwicklung übernahm Frank Goward und nach dessen Tod 1954 John Bertram Adams.
1955 war er wieder in Norwegen, wo er sich wieder der Kerntechnik widmete. Er leitete den Bau eines Kernreaktors in Halden und war 1957 bis 1958 Gründungsdirektor von Noratom. Später widmete sich Dahl unter anderem Raketen für wissenschaftliche Zwecke und war 1961 bis 1966 Vorstand des norwegischen Komitees für Weltraumforschung. 1968 ging er in den Ruhestand.
Sowohl die norwegischen Spezialisten für Teilchenbeschleuniger Björn Wiik (DESY) als auch Kjell Johnsen (CERN) wurden durch ihn gefördert. Johnsen charakterisierte Dahl so: *Er akzeptierte nur Herausforderungen und seine Intuition täuschte ihn nie.*[2]